FUNDAMENTALS OF ELECTROMAGNETIC FIELDS – February 12th, 2018

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Name and surname
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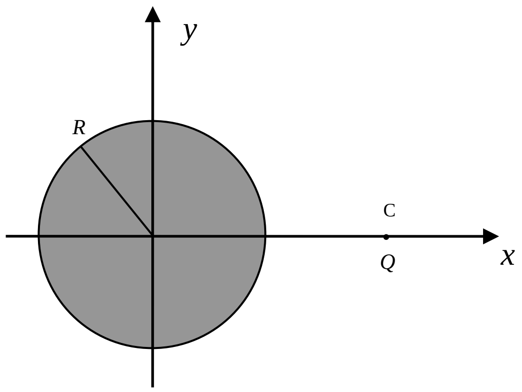
Problem 1 - A positive uniform charge density $\rho = 10^{-9}/(9\pi)$ C/m^3 is embedded in a dielectric sphere with radius R = 3 m (grey part in the figure). A negative point charge, with value $Q = -1$ nC, is fixed at point C(2R,0). Calculate the position of point $P(x_P,0)$ for which the total electric field is zero (consider only $x > R$).



Solution

The charge in the sphere is positive and the one in C is negative, which means that the two electric fields, considering only the portion of the plane for $x > R$, will be opposite only beyond C, i.e. for $x > 2R$. In this case, by applying the Gauss' theorem to the sphere:

$$\int_S \vec{D}_1 \cdot d\vec{S} = \int_V \rho\, dV = Q_{TOT}\,,$$

we obtain:

$$\vec{E}_1(x) = \frac{|Q_{TOT}|}{4\pi\varepsilon_0 x_P^2}\vec{\mu}_x$$

As ρ is uniform throughout the sphere:

$$Q_{TOT} = \int_V \rho\, dV = \rho\frac{4}{3}\pi R^3 = 4 \text{ nC}$$

For $x > 2R$, considering the charge in C, we obtain:

$$\vec{E}_2(x) = -\frac{|Q|}{4\pi\varepsilon_0 (x_P - 2R)^2}\,\vec{\mu}_x$$

Summing up the effects and searching for the zero of the electric field:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = 0 \quad \rightarrow \quad \frac{|Q_{TOT}|}{4\pi\varepsilon_0 x_P^2} = \frac{|Q|}{4\pi\varepsilon_0 (x_P - 2R)^2}$$

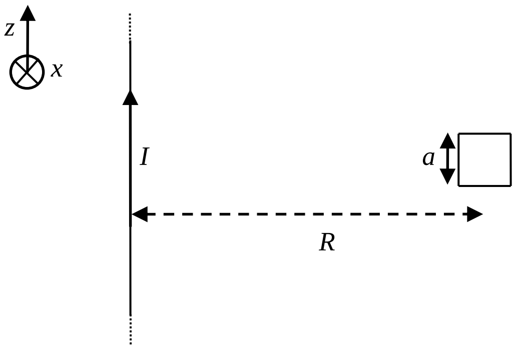
Solving for x, we obtain two solutions, i.e. $x_{P1} = 4R$ and $x_{P2} = \frac{4}{3} R$. The fist solution is acceptable, the second is not, as x_{P2} falls between the sphere and point C.

Problem 2 - Consider a square metallic ring with lateral dimension a = 1 cm and the electric current I flowing along a straight metallic wire of indefinite length. The temporal trend of the current is:

$$I = 100\sin(100\pi t)\,\vec{\mu}_z \text{ A} \quad \text{for} \quad t \geq 0 \text{ s}$$

The distance between wire and the metallic ring is R = 1 m. Calculate the electromotive force for $t \geq 0$ s. Assuming then that the metallic ring is associated to a resistance R = 10 Ω, calculate the value of the current flowing in the wire.

Assumption: given $R >> a$, the magnetic field generated by I can be considered to be constant for any point inside the metallic ring.



Solution

The magnetic field generated by the wire and flowing across the metallic ring is:

$$\vec{H} = \frac{I}{2\pi R}\vec{\mu}_x \quad \text{(A/m)}$$

The magnetic flux is given by (assuming the magnetic field is constant inside the ring):

$$\phi = \int_S \vec{B}\cdot dS = \mu_0 \vec{H}\,A = \mu_0 \vec{H}\,a^2 \quad \text{(Wb)}$$

So the electromotive force is:

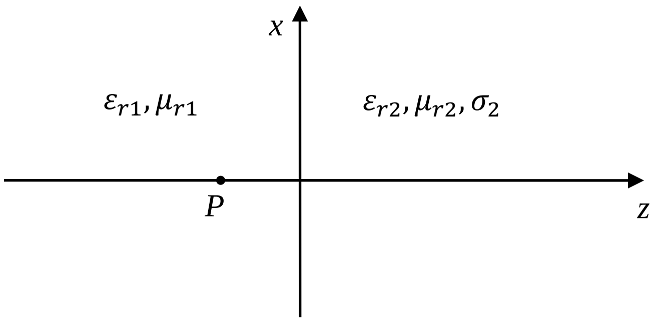
$$V(t) = \left| -\frac{\partial \phi}{\partial t} \right| = \begin{cases} \mu_0 \left[\frac{100\cos(100\pi t)\,100\pi}{2\pi R} \right] a^2 = 0.5\cos(100\pi t) & t \geq 0\,s \\ 0 & t < 0\,s \end{cases} \quad \text{(V)}$$

Thus the current is:

$I(t) = V(t)/R = 0.05\cos(100\pi t)$ A

Problem 3 - A uniform plane wave (frequency f = 20 GHz) propagates in a dielectric material ($\varepsilon_{r1} = 81, \mu_{r1} = 4, \sigma_1 = 0$ S/m) and impinges on another dielectric material ($\varepsilon_{r2} = 4$, $\mu_{r2} = 1, \sigma_2 = 10^{-2}$ S/m). The power density of the incident wave is S_i = 23.923 mW/m^2 and the polarization of the wave is linear along $\vec{\mu}_x$. Calculate the total electric field in $P(x = 0, y = 0, z = -\lambda_1)$.

Assume that the angle of the incident electric field $\vec{E}_i$ in (0,0,0) is zero.



Solution

First, let us calculate the intrinsic impedance for the two media. For the first one:

$$\eta_1 = \eta_0 \sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}} \approx 83.8\ \Omega$$

For the second one, the loss tangent is $\frac{\sigma}{\omega\varepsilon} \approx \mathbf{0.0022}$ << 1. Therefore the second medium can be well approximated as a good dielectric. Therefore:

$$\eta_2 = \eta_0 \sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} = 188.5\ \Omega$$

The reflection coefficient is:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \approx 0.385$$

The wavelength in the first medium is:

$$\lambda_1 = \frac{c}{f\sqrt{\varepsilon_{r1}\mu_{r1}}} = 0.83 \text{ mm}$$

The absolute value of the incident electric field is obtained from the incident power density:

$$S_i = \frac{1}{2}\frac{\left|\vec{E}_i\right|^2}{\eta_1} \rightarrow \left|\vec{E}_i\right| = 2 \text{ V/m}$$

Thus (considering the assumption):

$$\vec{E}_i(0,0,0) = 2\vec{\mu}_x \text{ V/m}$$

Let us calculate the propagation constant for the first medium:

$$\gamma_1 = j\beta_1 = j\frac{2\pi}{\lambda_1} = =7539.8 \text{ rad/m}$$

The incident and reflected fields are:

$$\vec{E}_i(z) = \vec{E}_i(0,0,0)e^{-j\beta_1 z}\vec{\mu}_x \quad \text{V/m}$$

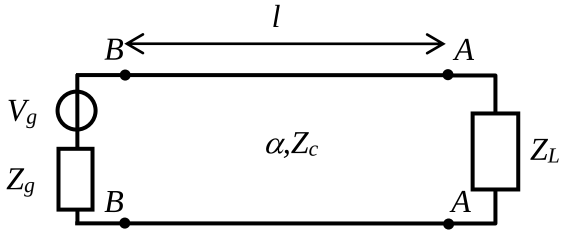
$$\vec{E}_r(z) = \vec{E}_i(0,0,0)\Gamma e^{j\beta_1 z}\vec{\mu}_x \quad \text{V/m}$$

The total electric field in *P* is therefore:

$$\vec{E}_t(P) = \vec{E}_i(P) + \vec{E}_r(P) = \vec{E}_i(0,0,0)e^{j\beta_1\lambda_1}\vec{\mu}_x + \vec{E}_i(0,0,0)\Gamma e^{-j\beta_1\lambda_1}\vec{\mu}_x = 2.77\vec{\mu}_x$$

Problem 4 - A source with V_g = 50 V and internal impedance Z_g = 50 Ω is connected to a load Z_L by a transmission line with characteristic impedance Z_C = 50 Ω. The line length is l = 30 m, the attenuation constant is α = 20 dB/km and the frequency is f = 300 MHz. Calculate the power absorbed by the load and lost along the line, in two cases:

a) The load is Z_L = 50 Ω
b) The load is a short circuit



Solution

a) The attenuation constant is first converted into Np/m:

$$\alpha' = \frac{\alpha}{8.686 \cdot 1000} = 2.3 \cdot 10^{-3} \text{ Np/m}$$

In this case, there is full match in the circuit. Thus, the power absorbed by the load is given by:

$$P_d = \frac{|V_g|^2}{8Z_g} = 6.25 \text{ W}$$

$$P_L = P_d e^{-2\alpha' l} \approx 5.44 \text{ W}$$

The power crossing section BB is:

$$P_{BB} = P_d$$

Finally, the power lost on the line is:

$$P_{line} = P_{BB} - P_L = 0.81 \text{ W}$$

b) A short circuit corresponds to Z_L = 0 Ω: in this case, there is high mismatch at the load section, as no power can be absorbed by a short circuit → $P_L = 0$ W. To calculate the power lost on the line, we need to find the input impedance at section BB.

$$\Gamma_L = \frac{Z_L - Z_C}{Z_L + Z_C} = -1$$

$$\beta = \frac{\omega}{c} = 6.28 \text{ rad/m}$$

$$\Gamma_{BB} = \Gamma_L e^{-j2\beta l} e^{-2\alpha l} = -0.871$$

As $Z_C = Z_g \rightarrow \Gamma_g = \Gamma_{BB}$

$$P_{BB} = P_d (1 - |\Gamma_g|^2) = 1.51 \text{ W}$$

All the power crossing section BB is lost along the line; in fact:

$$P_{line} = P_{BB} - P_L = P_{BB} = 1.51 \text{ W}$$