FUNDAMENTALS OF ELECTROMAGNETIC FIELDS – January 31st, 2018

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Name and surname
Identification number
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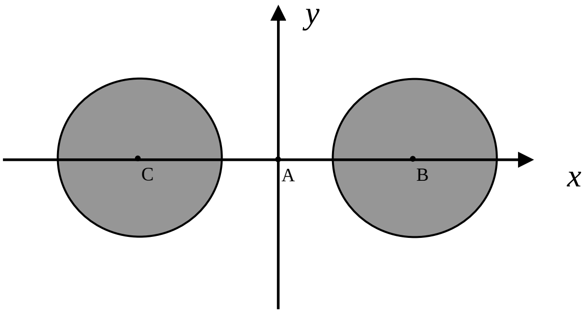
Problem 1 - A dialectric sphere, with radius R = 4 m and centered in B, is charged with a uniform density ρ = 10^{-10} C/m^3; another identical sphere, centered in C, is charged with a uniform density ρ = -10^{-10} C/m^3. Calculate the value of the electric field in A(0,0), B(1.5R,0) and in C(-1.5R,0)



Solution

Based on the Gauss' theorem:

$$\int_S \vec{D}\cdot d\vec{S} = \int_V \rho\, dV$$

we can obtain the contribution of both charged spheres in A. Given the sign of the charges and the geometry, both electric fields will be directed as $-\vec{\mu}_x$. For the sphere in B:

$$\varepsilon_0 \left|\vec{E}_A^{'}\right| 4\pi \left(x_B\right)^2 = \rho \frac{4}{3}\pi \left(R\right)^3 = Q_T = 2.681\cdot 10^{-8}\ C \quad \Rightarrow \quad \vec{E}_A^{'} = -\frac{Q_T}{4\pi\varepsilon_0 \left(x_B\right)^2}\vec{\mu}_x = -6.69\,\vec{\mu}_x \text{ V/m}$$

For the sphere in C:

$$\varepsilon_0 \left|\vec{E}_A^{''}\right| 4\pi \left(x_C\right)^2 = Q_T \quad \Rightarrow \quad \vec{E}_A^{''} = -\frac{Q_T}{4\pi\varepsilon_0 \left(x_C\right)^2}\vec{\mu}_x = -6.69\,\vec{\mu}_x \text{ V/m}$$

The total electric field in A is:

$$\vec{E}_A = \vec{E}_A^{'} + \vec{E}_A^{''} = -13.38\,\vec{\mu}_x \text{ V/m}$$

In B, according to the Gauss' theorem, the contribution of the sphere with center in B is zero, while the one due to the sphere in C is:

$$\varepsilon_0 \left|\vec{E}_B\right| 4\pi \left(x_C + x_B\right)^2 = Q_T \quad \Rightarrow \quad \vec{E}_B = -\frac{Q_T}{4\pi\varepsilon_0 \left(x_C + x_B\right)^2}\vec{\mu}_x = -1.67\,\vec{\mu}_x \text{ V/m}$$

Similarly, in C, according to the Gauss' theorem, the contribution of the sphere with center in C is zero, while the one due to the sphere in B is:

$$\varepsilon_0 \left|\vec{E}_C\right| 4\pi \left(x_C + x_B\right)^2 = Q_T \quad \Rightarrow \quad \vec{E}_C = -\frac{Q_T}{4\pi\varepsilon_0 \left(x_C + x_B\right)^2}\vec{\mu}_x = -1.67\,\vec{\mu}_x \text{ V/m}$$

Problem 2 - Consider two wires of indefinite length, where the following currents flow:

$$I_1 = \cos(100\pi t)\vec{\mu}_x \quad \text{A}$$

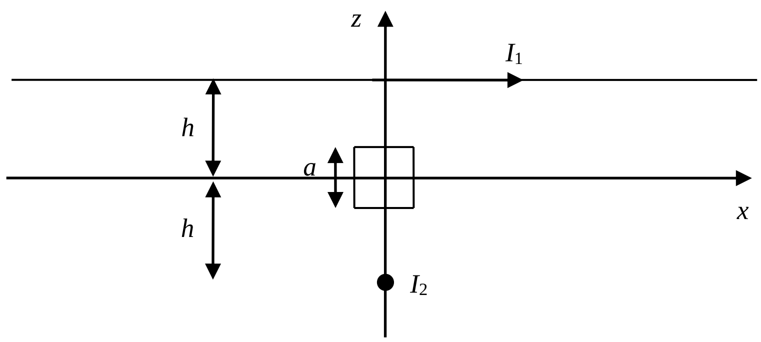
$$I_2 = -\cos(100\pi t)\vec{\mu}_y \quad \text{A}$$

Making reference to the figure below, h = 1 m, while a = 1 cm. Calculate:

1) The magnetic field generated by I_1 and by I_2 at the origin (0,0,0).

2) The current flowing in the metallic ring.

Assume then that the metallic ring is associated to a resistance R = 50 Ω and that, given $h >> a$, the magnetic field generated by both currents can be considered to be constant at any point inside the metallic ring.



Solution

1) The magnetic field generated by the two wires at (0,0,0) is:

$$\vec{H}_1 = \frac{|I_1|}{2\pi h}\vec{\mu}_y \text{ A/m} \quad \text{and} \quad \vec{H}_2 = -\frac{|I_2|}{2\pi h}\vec{\mu}_x \text{ A/m}$$

2) The contribution to the magnetic flux is only given by I_1 as $\vec{H}_2$ is parallel to the ring. Therefore:

$$\phi = \int_S \vec{B}\cdot dS = \mu_0 \left|\vec{H}_1\right| A = \mu_0 \left|\vec{H}_1\right| a^2 = \mu_0 \frac{|I_1|}{2\pi h} a^2 \quad \text{Wb}$$

Therefore the electromotive force is:

$$V(t) = \left|-\frac{\partial \phi}{\partial t}\right| = \left|-\frac{\partial}{\partial t}\left(\mu_0 \frac{\cos(100\pi t)}{2\pi h} a^2\right)\right| = \frac{\mu_0 a^2}{2\pi h}\left|-\frac{\partial}{\partial t}\left(\cos(100\pi t)\right)\right| = \frac{50\mu_0 a^2}{h}\sin(100\pi t) \qquad \text{V}$$

Thus the current generated in the wire is:

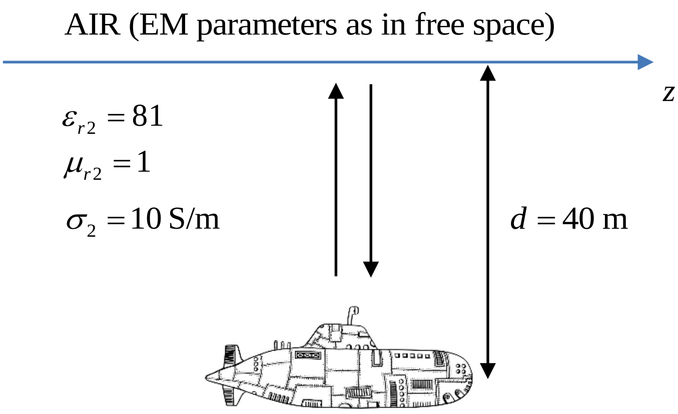
$$I(t) = V(t)/R = \frac{\mu_0 a^2}{h} \sin(100\pi t) \text{ A}$$

Problem 3 - A submarine transmits electromagnetic pulses towards the sea surface to keep track of its depth. Assuming that the submarine emits plane waves with electric field $\left|\vec{E}_{out}\right| = 5$ V/m at frequency f = 1 kHz, and that its depth is d = 40 m, calculate:

1) The wavelength underwater.

2) The propagation velocity underwater.

3) The power density reaching back the submarine after reflection on the sea surface.



Solution

1) First we need to characterize the electromagnetically the first medium (sea water). In this case, the loss tangent is $\frac{\sigma}{\omega\varepsilon}$ >> 1. Therefore the second medium can be well approximated as a good conductor. Therefore the attenuation and propagation constants are:

$$\alpha = \beta = \sqrt{\pi f \mu \sigma} = 0.1987 \text{ 1/m}$$

As for the intrinsic impedance, we obtain:

$$\eta_1 = \sqrt{\frac{\pi f \mu}{\sigma}}(1+j) = 0.0199(1+j)\ \Omega$$

The wavelength is:

$$\lambda = \frac{2\pi}{\beta} = 31.62 \text{ m}$$

2) The propagation velocity is:

$$v = \frac{\omega}{\beta} = 3.162 \times 10^4 \text{ m/s}$$

3) For the first medium (air/free space), η_2 = 377 Ω. The reflection coefficient is therefore:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \approx 1$$

The power density emitted by the submarine is:

$$S_{out} = \frac{1}{2}\frac{\left|\vec{E}_{out}\right|^2}{|\eta_1|}\cos(\measuredangle \eta_1) = 314.6 \text{ W/m}^2$$

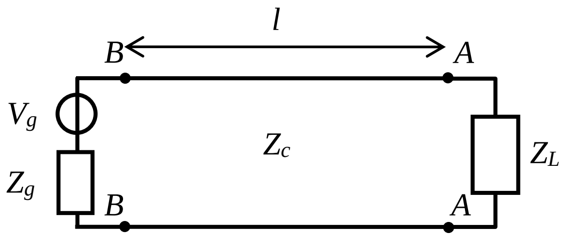
The power density reaching the submarine after reflection is:

$$S_{back} = S_{out}e^{-2\alpha d}\left|\Gamma\right|^2 e^{-2\alpha d} = \frac{1}{2}\frac{\left|\vec{E}_{out}\right|^2}{|\eta_1|}\cos(\measuredangle \eta_1)e^{-4\alpha d}\left|\Gamma\right|^2 = 4.9 \text{ pW/m}^2$$

<u>Problem 4</u> - A source with voltage V_g = 10 V and internal impedance Z_g = 50 Ω is connected to a load Z_L = -j10 Ω by a transmission line with characteristic impedance Z_C = 100 Ω. The line length is l = 6.25 m and the frequency is f = 300 MHz.

Calculate:

1) The power absorbed by the load.
2) The voltage at the beginning of the line (section BB) V_{BB}.
3) The value of Z_L to maximize P_L, the power transferred to the load.
4) The value of P_L for the conditions at point 3).



Solution

1) As the load is imaginary (it corresponds to a capacitor), no power will be absorbed by Z_L.

2) The wavelength is:

$$\lambda = \lambda_0 = c/f = 1 \text{ m}$$

The length of the line l corresponds to:

$$\lambda = 6\lambda + \lambda/4$$

Therefore, the input impedance is simply given by:

$$Z_{BB} = \frac{Z_C^2}{Z_L} = j1000 \ \Omega$$

The voltage at the beginning of the line is found as:

$$V_{BB} = V_g \frac{Z_{BB}}{Z_{BB} + Z_g} = 9.9751 + j0.4988 \text{ V}$$

3) To maximize the power transfer, Z_{BB} needs to be equal to Z_g:

$$Z_{BB} = \frac{Z_C^2}{Z_L} = Z_g = 50 \ \Omega \ \rightarrow \ Z_L = \frac{Z_C^2}{Z_g} = 200 \ \Omega$$

4) For the conditions at point 3), $\Gamma_g = 0$ and therefore $P_L = P_d = \dfrac{|V_g|^2}{8\text{Re}\{Z_g\}} = 0.25$ W