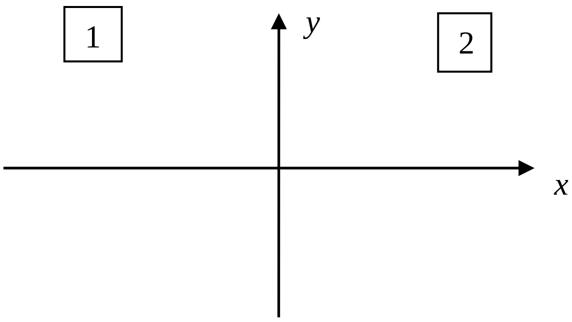
FUNDAMENTALS OF ELECTROMAGNETIC FIELDS – July 8th, 2019

1	**2**	**3**	**4**

Name and surname
Identification number
Signature

Problem 1 - Making reference to the figure below depicting two perfect dielectrics, the first medium is characterized by ε_{r1} = 4 and μ_{r1} = 4, while the second material is characterized by ε_{r2} = 6 and μ_{r2} = 1. A uniform current density $\vec{J}_S = -5\vec{\mu}_z$ A/m flows along the *zy* plane, and a uniform charge density σ_S = -10^{-12} C/m^2 is embedded in such a plane as well. The electric field and the magnetic field in medium 1 are $\vec{E}_1 = 2\vec{\mu}_y$ V/m and $\vec{H}_1 = -4\vec{\mu}_x$ A/m, respectively. Calculate the electric field and the magnetic field in the second medium.



Solution

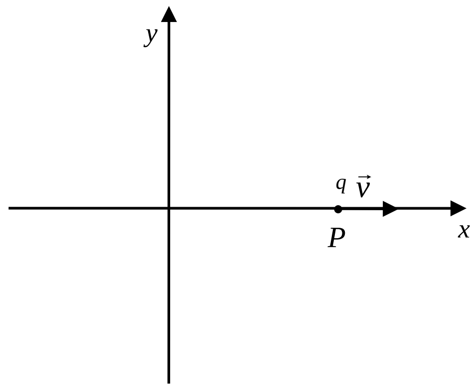
The electric field in the second medium can be calculated by resorting to the boundary condition for components tangent to the boundary between the two media, i.e.:

$$E_{1y} = E_{2y} \rightarrow E_{2y} = 2 \text{ V/m}$$

The magnetic field can be calculated by resorting to the boundary condition for components orthogonal to the boundary between the two media, i.e.:

$$B_{1x} = B_{2x} \rightarrow \mu_0\mu_{r2}H_{2x} = \mu_0\mu_{r1}H_{1x} \rightarrow H_{2x} = \frac{\mu_{r1}}{\mu_{r2}}H_{1x} = -16 \text{ A/m}$$

Problem 2 - An electron with charge q = -1.6·10^{-19} C is in P(2,0), travelling with velocity $\vec{v} = c\,\vec{\mu}_x$ m/s, being c the speed of light. In addition, there is a magnetic field $\vec{H} = 3\vec{\mu}_z$ mA/m, which is uniform everywhere. The magnetic permeability μ_r = 4. Determine the position on the xy plane of a point charge Q = 1 nC such that the total force acting on the electron is zero.



Solution

The electron will be subject to two forces: one due to the electric field (Coulomb's force) and the other one due to the magnetic field (Lorentz's force).

The Lorentz's force depends on the uniform magnetic field:

$$\vec{F}_L = q\vec{v} \times \mu_r \mu_0 \vec{H} = -q\mu_r\mu_0 c\left|\vec{H}\right|\vec{\mu}_y = 7.2 \cdot 10^{-19}\,\vec{\mu}_y \text{ N}$$

The additional charge Q can counteract the effect of the Lorentz's force (Coulomb's force)

$$\vec{F}_C = q\vec{E} \text{ N}$$

The electric field generated by the charge Q in P(2,0) is:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{\mu}_{qQ} \text{ V/m}$$

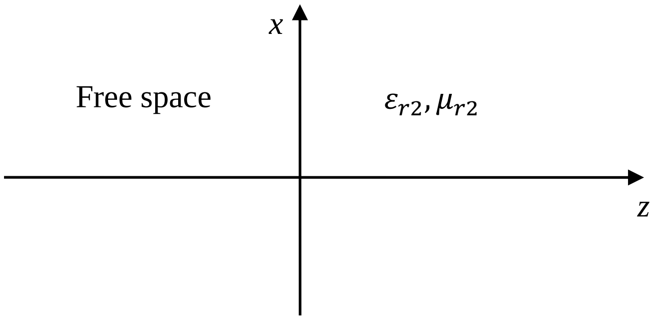
where r is the distance between the two charges and $\vec{\mu}_{qQ}$ is the versor in the direction connecting the two charges. As Q is positive and q is negative, Coulomb's force will be attractive: to counteract $\vec{F}_L$, Q must be placed on the xy plane in K(2,-r). r can be determined by imposing that:

$$\left|\vec{F}_C\right| = \left|\vec{F}_L\right| \quad \Rightarrow \quad \frac{|qQ|}{4\pi\varepsilon_0 r^2} = \left|\vec{F}_L\right| \quad \Rightarrow \quad r = \sqrt{\frac{|qQ|}{4\pi\varepsilon_0\left|\vec{F}_L\right|}} = 1.41\,\text{m}$$

Problem 3 - A uniform plane wave with vertical polarization (frequency f = 300 MHz) propagates along z, from free space into a medium with the following electromagnetic features: ε_{r2} = 1-j, μ_{r2} = 1. The amplitude of the incident electric field is E_0 = 10 V/m.

Calculate:

a) The wavelength in the second medium
b) The expression of the magnetic field associated to the reflected wave



Solution

a) The wavelength in the second medium depends on the phase constant, which can be determined using the following expression (no approximations are possible as the loss tangent is equal to 1):

$$\gamma_2 = \sqrt{j\omega\mu_2 j\omega\varepsilon_2} = 2.86 + j6.91 \text{ 1/m}$$

Therefore, the wavelength is:

$$\lambda_2 = \frac{2\pi}{\beta_2} = 0.91 \text{ m}$$

b) To calculate the reflected wave, it is first necessary to find the reflection coefficient:

$$\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \eta_0 \approx 377\ \Omega$$

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{j\omega\varepsilon_2}} = 292.7 + j121.2\ \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -0.09 + j0.197$$

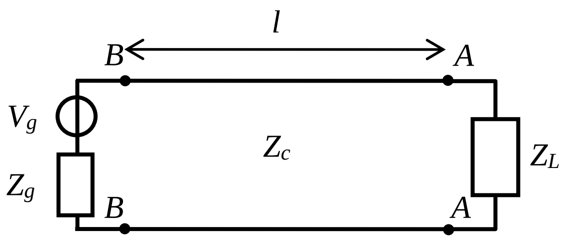
Assuming that the incident electric field is along x (one of the possible vertical polarizations), the reflected magnetic field is:

$$\vec{H}_r(z) = -\Gamma\vec{\mu}_y \frac{E_0}{\eta_0} e^{j\beta_1 z} = \vec{\mu}_y (2.4 \text{ - } j5.2) e^{j6.28z} \text{ mA/m}$$

Problem 4 - A source with voltage V_g = 50 V and internal impedance Z_g = 100 Ω is connected to a load Z_L = 25 Ω by a transmission line with characteristic impedance Z_C = 50 Ω, the frequency is f = 300 MHz and the length of the line is l = 4.25 m

Calculate:

a) The power absorbed by the load
b) The temporal trend of the voltage at the beginning of the line (section BB below), V_B



Solution

a) The wavelength is:

$$\lambda = \lambda_0 = c/f = 1 \text{ m}$$

As a consequence, the length of the line normalized to the wave length is:

$$l/\lambda = 4.25 = 4 + 0.25 \text{ m}$$

In other terms, the line is a $\lambda/4$.

As a result, the load at section BB can be easily calculated as:

$$Z_{BB} = \frac{Z_C^2}{Z_L} = 100 \ \Omega$$

The reflection coefficient for the source is:

$$\Gamma_g = \frac{Z_{BB} - Z_g}{Z_{BB} + Z_g} = 0$$

Therefore, the power absorbed by the load is:

$$P_L = P_{AV} = \frac{|V_g|^2}{8\,\mathrm{Re}[Z_g]} = 3.125 \text{ W}$$

b) The voltage at the beginning of the line is:

$$V_{BB} = V_g \frac{Z_{BB}}{Z_{BB} + Z_g} = 25 \text{ V}$$

c) The trend of V_{BB} in time is given by:

$$v_{BB}(t) = \mathrm{Re}\left[V_{BB}\, e^{j\omega t}\right] = 25\cos(\omega t) \;\; \text{V}$$