POLITECNICO MILANO 1863 Academic year 2022/2023	094784 **Fund. of Electromagnetic Fields**	Date: ______________________
	Last Name:___________________	First Name: _________________
	Personal code ("Persona"): ______________________________	Signature: ______________________________

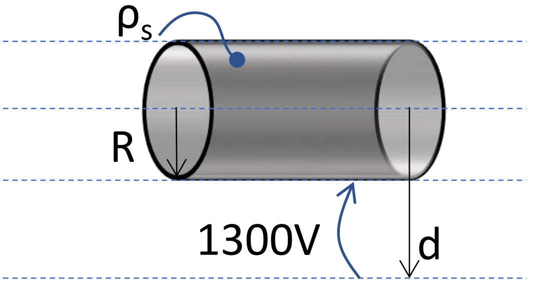
16 January 2023 EXAM

Please answer the following questions/problems, providing a meaningful explanation of the steps/computations involved. Please specify units for all numeric results requiring them, otherwise those results will be considered wrong. Allowed support material: books, notes, scientific calculator.

Exercise 1 [8 points]

A surface charge density ρ_s=10^{-6}C/m^2 uniformly covers an infinitely-long cylinder with radius R=1cm in a homogeneous medium with unknown relative dielectric constant ε_r.
Knowing that the electrostatic potential at a distance d=10cm from the center is 1300V lower than on the cylinder surface (with radius R):

1. starting from Gauss theorem, compute the dielectric constant of the medium surrounding the charge density
2. compute the electric field at the center of the charged cylinder



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A plane wave in phasor form has an electric field which is directed along $\hat{\imath}_x$ with magnitude 10V/m and a magnetic field directed along $\hat{\imath}_y$ with magnitude 0.053A/m, both with phase 0°.
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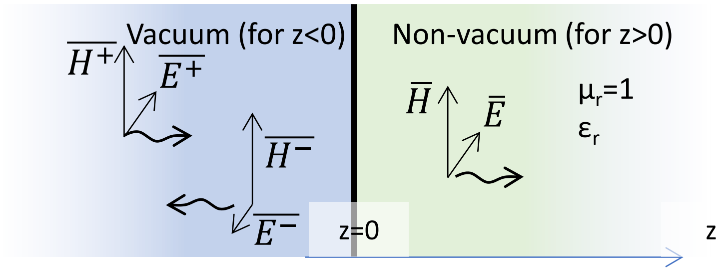

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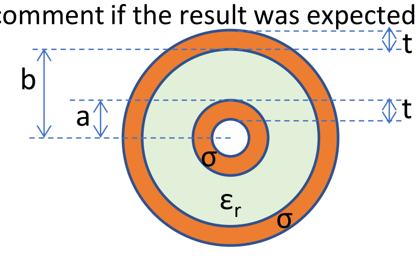

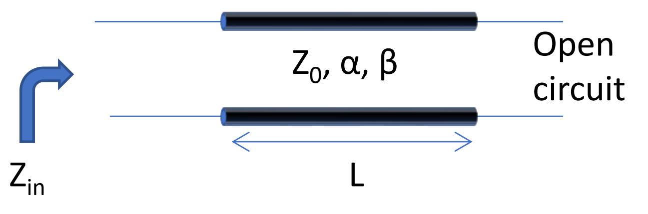

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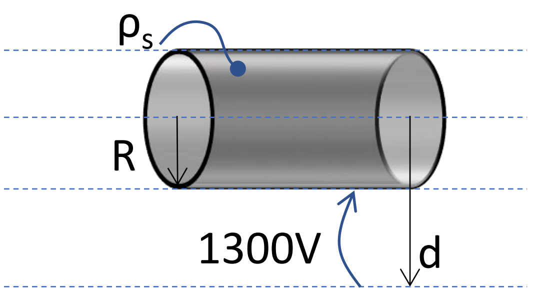

Solution

Thanks to symmetry, the electrostatic field outside the cylinder depends only on the distance from the center *r* and the electric field has only radial component E_r, and so does the electric flux density $D=D_r(r)\,\hat{\imath}_r$, in cylindrical coordinates.
Applying Gauss theorem on a cylindrical surface S of radius *d*, centered on the cylinder, larger than the cylinder (r>R) and of finite length h:

$$\oiint_S \overline{D}\cdot\overline{dS} = \iiint_V \rho_V dV$$

$$D_r(r)\int_0^h\int_0^{2\pi} r \quad d\varphi\, dz = D_r(r)2\pi r h = \rho_s 2\pi R h$$

$$\overline{D}(d) = \rho_s \frac{R}{r}\widehat{\imath_r}$$

The electric field is hence E=D/ε, but $\varepsilon=\varepsilon_0\varepsilon_r$ and ε_r is unknown.
The electrostatic potential variation moving from a point on the cylinder surface (R) to a point further away (d) is:

$$\Delta V_{R\to d} = -\int_R^d \bar{E}\circ dr\,\hat{\imath}_r = -\rho_s\frac{R}{\epsilon}\int_R^d \frac{1}{r}dr = -\rho_s\frac{R}{\epsilon}\ln\left(\frac{d}{R}\right) = -1300V$$

Hence epsilon can be found to be $\varepsilon=1.771\cdot 10^{-11}$ F/m= $\varepsilon_0\varepsilon_r$, from which $\varepsilon_r=2$.
At the center of the cylinder (r=0), Gauss theorem states that the electric flux density due to the charged cylinder is 0, since the enclosed charge is null. Hence the electric field is also null.

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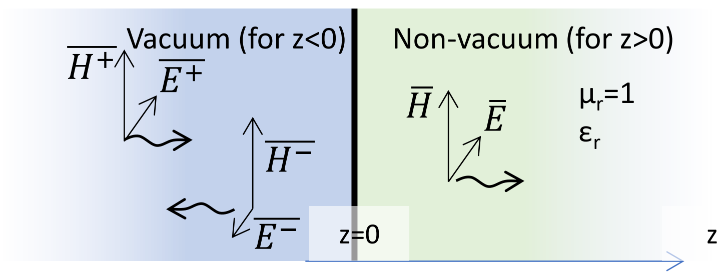

Solution

The wave impedance for a single plane wave is merely the ratio of the electric and magnetic field phasors amplitude: $\bar{E}$=10V/m $\hat{\imath}_x$; $\bar{H}$=0.053A/m $\hat{\imath}_y$. Thus η = 10V/m/(0.053A/m)=188.7Ω.
In a lossless medium, the wave impedance is given by η=sqrt(μ/ε)=37 Ω sqrt(μ_r/ε_r)
from which ε_r=4.

The real power density carried by a plane wave and the propagation direction are given by the Poynting vector:
$\bar{S}$ =1/2 Real($\bar{E}$ x $\bar{H}^*$)=0.5 10V/m 0.053A/m $\hat{\imath}_x$ x $\hat{\imath}_y$ =0.265W/m² $\hat{\imath}_z$

In normal incidence, the reflection coefficient relating the impinging and reflected electric fields at the discontinuity between the two media (medium 1 where the impinging and reflected waves exist, 2 where only the transmitted wave exists) is $\Gamma=(\eta_2-\eta_1)/(\eta_2+\eta_1)$.
The electric field transmitted is related to the impinging one by the transmission coefficient T=1+Γ
Since the medium in z<0 is air, the wave impedance is η_1 =377 Ω, whereas the impedance η=188.7Ω computed earlier is η_2.
Then Γ=-0.33 and T=0.67.
Since the known electric field $\bar{E}$ is the transmitted one (evaluated at the discontinuity), the impinging one at z=0 is merely:
$\bar{E}^+ = \frac{\bar{E}}{T} = 14.9\frac{V}{m}\hat{\imath}_x$ and the magnetic field is, following $\eta_1 = E_x{}^+/H_y{}^+$, $\bar{H}^+= 40\frac{mA}{m}\hat{\imath}_y$
whereas the reflected one in z=0 is $\bar{E}^- = \Gamma\bar{E}^+$=-4.9 V/m $\hat{\imath}_x$ and $-\eta_1 = E_x{}^-/H_y{}^-$: $\bar{H}^-= 13\frac{mA}{m}\hat{\imath}_y$

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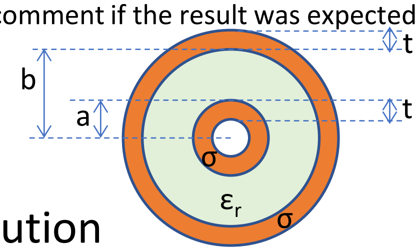

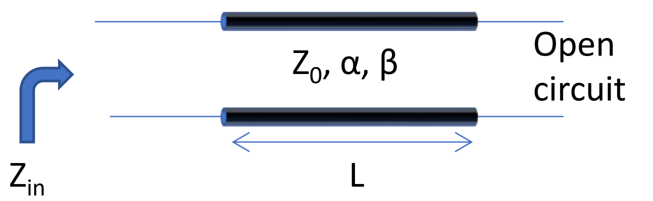

Solution

In the quasistatic approximation for a TEM line, L and C per unit length are related:
$L\,C=\varepsilon\,\mu \rightarrow C=\varepsilon\,\mu\,/\,L$ where L=277nH/m and thus C=110.4pF/m.
Using the low-loss approximation ($R \ll \omega L$, $G \ll \omega C$): Z_0=sqrt(L/C)=50Ohm
Ohmic loss in the conductors can be modeled by considering that current will flow only through an annulus of depth δ (skin depth) = sqrt(2/(ω μ σ))=1.3μm. This is much smaller than the 0.5mm conductor thickness and thus the skin depth is responsible for limiting the current flow.
The surface resistance is thus Rs=1/(δσ)=0.077 Ω/□ and the resistance of each conductor is:
$R_a=1/(\sigma\,\delta\,2\,\pi\,a)$=Rs/perimeter$_a$=12.2Ω/m; $R_b=1/(\sigma\,\delta\,2\,\pi\,b)$=Rs/perimeter$_b$=3.06Ω/m
The total resistance per unit length of the transmission line is hence R=Ra+Rb=15.3Ω/m.
Dielectric losses can be modeled by observing that the actual capacitance will depend on dielectric loss by means of the imaginary part: G=ω C $\tan_\delta$ where $\tan_\delta=10^{-3}$ and thus G=0.01S/m.
The attenuation constant and phase constant thus are:

$\alpha=G*Z_0/2+R/2/Z_0$=0.25Np/m + 0.153Np/m=0.403Np/m=3.5dB/m

$\beta=\omega$ sqrt(LC)=521.189rad/m → $\lambda=2\,\pi/\beta$=12mm → The line is L=20λ=24cm

Z_{in} depends on the refl. coeff. at the input of the line (Γ_{in}). This depends on the refl. coeff. at the load (Γ_L).
Since the load is an open circuit, Z_L=infinity and thus $\Gamma_L=(Z_L-Z_0)/(Z_L+Z_0)=+1$ is the ratio between the backward and the forward voltage waves at the load section.
The same ratio can be evaluated at the input of the line:
$\Gamma_{in}=\Gamma_L\,e^{-2\gamma L}=1\cdot e^{-2\beta L}\,e^{-2\alpha L}=e^{-2\alpha L}=0.824$
From which the input impedance at the input of the line is obtained $Z_{in}=Z_0(1+\Gamma_{in})/(1-\Gamma_{in})$=518 Ω.
Comment: a lossless line with length multiple of λ merely presents the same load impedance at its input. If such line is left open it would thus present infinite input impedance. Due to the non-zero attenuation constant α, this behavior is limited: only a very-high input impedance is in fact observed at the input. If the line were much longer, the attenuation would further act and the input impedance would tend toward Z0.