POLITECNICO MILANO 1863 Academic year 2022/2023	094784 **Fund. of Electromagnetic Fields**	Date: ______________________
	Last Name:____________________	First Name: __________________
	Student ID (“Persona” or “Matricola”): ______________________________	Signature: ______________________________

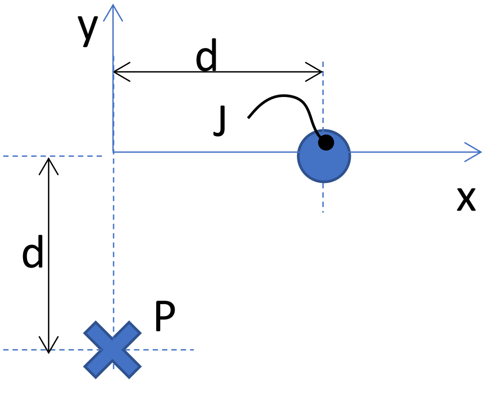
22 June 2023 EXAM

Please answer the following questions/problems, providing a meaningful explanation of the steps/computations involved. Please specify units for all numeric results requiring them, otherwise those results will be considered wrong. Allowed support material: books, notes, scientific calculator.

Exercise 1 [8 points]

A constant (static) uniform volumetric current density $\bar{J} = J\ \hat{\imath}_z$ flows along z in an infinite wire of radius R=2mm placed as in the figure, with d=15cm in a uniform material. The total magnetic field in the observation point P is: $\bar{H}$=(-15$\hat{\imath}_x + 15\ \hat{\imath}_y$)mA/m:

1. derive the expression of the magnetic field outside an infinite wire carrying a uniform volumetric current density J starting from Ampere’s law;
2. compute J and the total current I carried by the wire, specifying whether it is toward +z or –z.



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Two plane waves at a frequency of 12GHz counterpropagate in vacuum toward ±z with the electric field directed along x. The wave propagating toward +z carries a power density S^+=75.6mW/m^2. The wave propagating toward –z instead carries a power density of 8.3mW/m^2.

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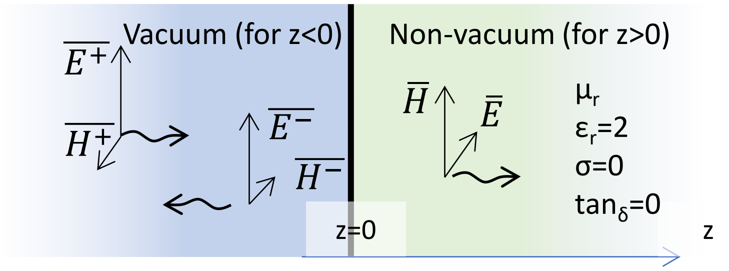

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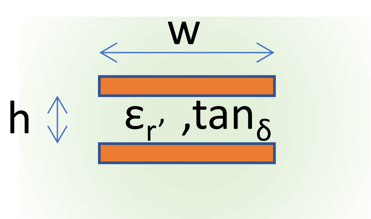

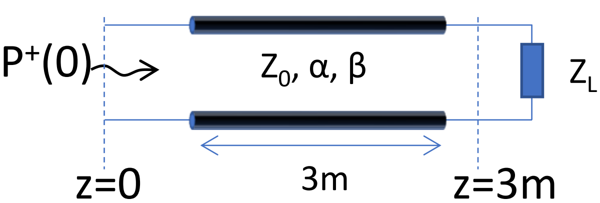

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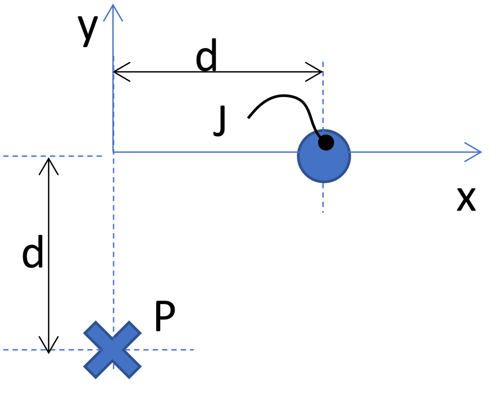


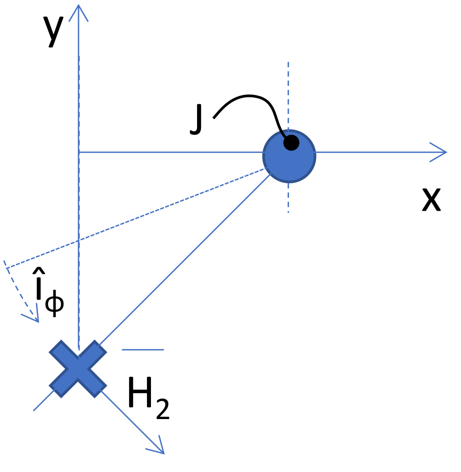
Solution

The expression of magnetic field due to a single wire can be obtained by circular symmetry: $\bar{H}$=$H_\phi(r)\hat{\imath}_\phi$ as being only directed along the ϕ versor in cylindrical coordinates around the wire's axis and from Ampere's law:

$$\oint_L \bar{H} \cdot \overline{dl} = \iint_{S\ enclosed\ by\ L} \bar{J} \cdot \overline{dS}$$

Choosing a circular path of radius *r>R* and if the uniform current density is parallel to the wire, $H_\phi(r)2\pi r = J\pi R^2 = I$.

The field from current distribution $\bar{J} = J\ \hat{\imath}_z$ creates a field in the observation point which has both $\hat{\imath}_x$ and $-\hat{\imath}_y$ components: $\bar{H}\,(P) = \frac{I}{2\pi d\,\sqrt{2}}\hat{\imath}_{\phi(P)} = \frac{I}{2\pi d\,\sqrt{2}}\frac{\hat{\imath}_x - \hat{\imath}_y}{\sqrt{2}}$ which is obtained by observing that the $\bar{H}$ field is directed at -45° with respect to the x axis.



The observed magnetic field exactly fits this form, and equating either the x or y component yields:

$\frac{I}{2\pi d\,\sqrt{2}}\frac{1}{\sqrt{2}} = -15\frac{mA}{m} \rightarrow I = -15\frac{mA}{m}4\,\pi d$= -28.3mA

Thus the total current carried by the wire is 28.3mA toward –z.

The corresponding uniform volumetric current density has thus a z component given by I/"cross section of wire":

J=-28.3mA/(πR^2)=2252 A/m^2

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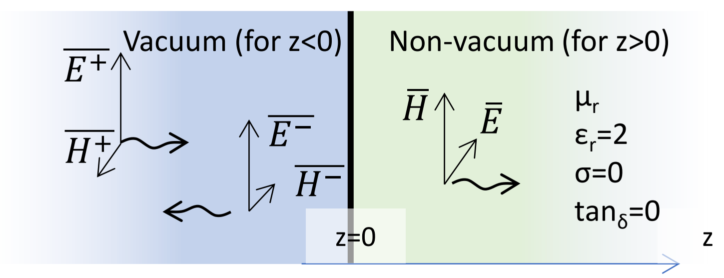

Solution

A plane wave in a lossless medium (such as vacuum in z<0) propagating toward +z with electric field vector E^+ along +x ($E^+=E^+_x\, i_x$) must have its magnetic field vector H^+ along +y ($H^+=H^+_y\, i_y$), such that the Poynting vector expressing the real power density carried is positive and directed toward +z:
S^+=0.5 Real(E^+ x H^{+*})=0.5 Real(E^+_x $(H^+_y)^*$)i_z =0.5Real(E^+_x $(E^+_x)^*/\eta_1$)=0.5|E^+_x|$^2/\eta_1$
Where the last step uses that the magnetic and electric fields are in phase and their magnitudes are governed by the real wave impedance of the material η_1=sqrt((μ_0 μ_{r1})/(ε_0 ε_{r1}))=377 Ω such that $E^+_x/H^+_y=\eta_1$.
Hence the electric field of the +z wave has amplitude |E^+_x|=7.55V/m.
Recalling that the phase of the wave toward +z is 0°, the electric field is:
$E^+(z)$=|E^+_x| i_x e^{j0} e^{-jk1z} with k1 the wave number in medium 1: k1=ω sqrt(μ_0 $\mu_{r1}\varepsilon_0$ ε_{r1})=251.5 rad/m.

The same can be done for the electric field E^-_x of the wave propagating toward –z, although observing that its magnetic field is H_y = H^-_y i_y with $E^-=E^-_x\, i_x$ and $H^-_y=-E^-_x/\eta_1$
Hence the electric field of the –z wave has amplitude |E^-_x|=2.5V/m and the electric field in vector phasor form is: $E^-(z)$=|E^-_x | i_x e^{j0} e^{+jk1z}

The expression of the total electric field as a function of time is thus:
$\mathcal{E}(z,t)$=real(($E^+(z) + E^-(z)$) $e^{j\omega t}$)=(7.55cos(ωt+0 rad –k_1z)+ 2.5cos(ωt+0 rad +k_1z))i_x V/m

In normal incidence, the reflected wave is related to the impinging one via the reflection coefficient: $E^-(0)=E^+(0)$ Γ, whose expression is Γ=(η_2 - η_1)/(η_2+η_1). With the values computed before, Γ=0.33.
Thus η_2= η_1 (1+Γ)/(1-Γ)=748 Ω.
In such a lossless medium, η_2=377 Ω*sqrt(μ_r/ε_r), from which μ_r=8.

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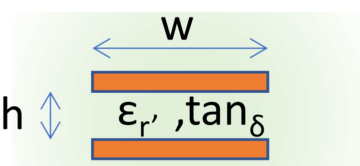

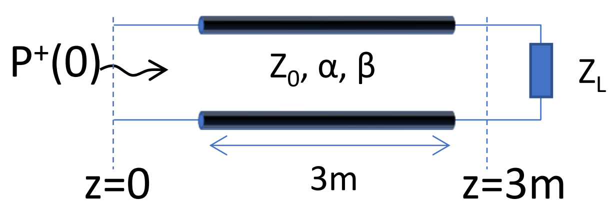

Solution

The capacitance of a parallel plate capacitor with area A=w L and gap h is C (non p.u.l.)=$\varepsilon_0\ \varepsilon_r$ A/h.
The capacitance per unit length of a parallel plate line is hence approximately C=$\varepsilon_0\ \varepsilon_r$ w/h.
The quasistatic approximation allows to use this expression also for computations at high frequency such as in the transmission line case we are considering. Therefore h= $\varepsilon_0\ \varepsilon_r$ w/C=2.2mm.

The low-loss approximation (R<< ωL, G<< ωC) allows to simplify the computation of the characteristic impedance and of the propagation constant of the line:
Z0≈sqrt(L/C)=50 Ω and γ=sqrt((j ω L+R)(j ω C+G))≈jωsqrt(LC)+R/(2Z_0)+GZ_0/2
From which β=imag(γ)=ω sqrt(LC)=695rad/m and α=real(γ)=GZ0/2=0.25Np/m=2.17dB/m

The power absorbed by the load can be computed by the difference between the power on the line travelling toward the load and the power on the line travelling back, both evaluated at the load section. However, the condition of ZL=Z0 implies that the reflection coefficient at the load section is equal to 0. Hence there is no counter-propagating wave.
Thus the power absorbed by the load is the whole power carried by the forward-propagating wave when it reaches the load.
Since the forward-propagating wave travels as $V^+(z)=V^+(0)\ e^{-\gamma z}$ (and similarly for the corresponding current), the power carried by the forward propagating wave is
$P^+(z) = 0.5 \cdot \mathrm{real}(V^+(0) \cdot \mathrm{conj}(I^+(0)))\ e^{-2\alpha z} = P^+(0)\ e^{-2\alpha z}$
The amount of power of the forward-travelling wave reaching the load and thus absorbed by it (with ZL=Z0) is thus 10W $e^{-2\ 0.25\mathrm{Np/m}\ 3\mathrm{m}}$=2.23W.