POLITECNICO MILANO 1863 Academic year 2022/2023	094784 **Fund. of Electromagnetic Fields**	Date: ______________________
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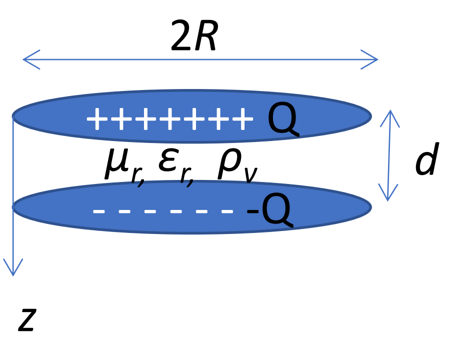
	Student ID ("Persona" or "Matricola"): ______________________________	Signature: ___________________________

07 July 2023 EXAM

Please answer the following questions/problems, providing a meaningful explanation of the steps/computations involved. Please specify units for all numeric results requiring them, otherwise those results will be considered wrong. Allowed support material: books, notes, scientific calculator.

Exercise 1 [8 points]

A parallel plate capacitor has capacitance of 0.2nF. The total charge on each of the plates is Q=0.2nC, and the plates are circular. The capacitor is filled with ABS (εr=3.1, μr=1). Fields can be assumed to be 0 outside the capacitor and any fringing fields can be neglected. Fields between the plates are constant.



1. Compute the spacing *d* and radius *R* of the plates so that the electrostatic field between the two plates is at most 0.01% of the dielectric strength of ABS (dielectric strength of ABS=15kV/mm). Compute the potential difference ΔV.
2. Compute the surface charge density ρ_S on the plates, assuming it to be uniform
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Exercise 2 [12 pt]

A plane wave at a frequency of 25GHz propagates in “Medium 1” from negative z toward +z with the electric field directed along x and the magnetic field along y. For this wave in z=0 , the electric field component phasor is 15V/m, whereas the magnetic field component phasor is 0.0563A/m.

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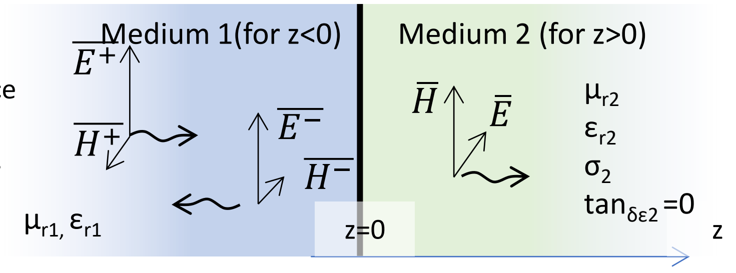

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A coaxial cable with a PEC conductors with inner radius a=1.2mm and outer radius b=3.6mm, filled with teflon (ε_r=2.04, $\tan_{\delta\varepsilon}$=0, σ=0, μ_r=1) carries a wave toward +z with power $P^+(0)$=500W at 18GHz.

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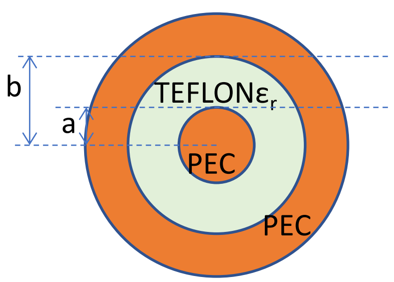

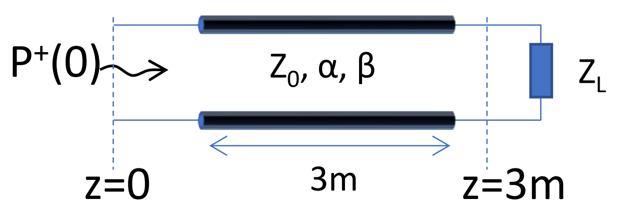

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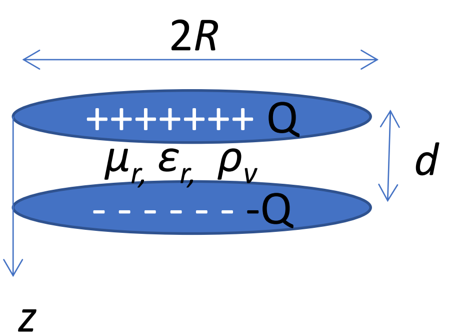

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Solution

The capacitance between two charged bodies (Q and –Q), by definition, is C=Q/ΔV.
In a parallel plate capacitor the electrostatic field is constant in the whole capacitor and is directed away from the positively-charged plate: E $\hat{\imath}_z$
The electrostatic potential between the two plates corresponding to this electric field is thus:
$\Delta V_{\{-Q\to+Q\}} = -\Delta V_{\{z=0\to z=d\}} = \int_0^d E\hat{\imath}_z\cdot\hat{\imath}_z dz = Ed$ and ΔV=Q/C=1V
Therefore, enforcing that E=15kV/mm/10000=1.5V/mm=1500V/m, d=Q/C/E=0.667mm.
The needed radius to obtain the required capacitance is obtained from the expression of capacitance.
For a parallel plate capacitor with circular plates the formula is:
C=π R^2 ε/d → R=sqrt(dC/π/ε0/εr)=3.93cm

Derivation (not needed):
Applying Gauss theorem to a cylindrical surface enclosing only the positive plate:
$\oiint_S \overline{D}\cdot\overline{dS} = \iiint_V \rho_V dV \quad \to \text{D} \int_0^R\int_0^{2\pi} r d\varphi\, dr = D\frac{R^2}{2}2\pi = DR^2\pi \quad = Q \ \to E = D/\varepsilon = Q/(\pi R^2\varepsilon)$
ΔV=Ed=Qd/(πR^2 ε) → C=Q/ΔV=(Qπ R^2 ε) /(Qd)=π R^2 ε/d

Surface charge density, assuming uniform distribution: ρ_s= Q/(πR^2)=41.2nC/m^2

Resistivity ρ is related to conductivity σ: σ=1/ρ=10^{-16}S/m. The leakage current is thus a current which flows through the capacitor while charged.
The relation between current density J and electric field is J=σE=0.15pA/m^2. J is hence directed as E, thus from the positively charged plate to the negatively charged one. No current flows outside of the plates due to the fields assumed 0.
The total leakage current (not required) is hence: I=J π R^2=0.73pA
This is confirmed by computing the total resistance of the ABS cylinder:
R=ρd/(πR^2)=1.37 · 10^{15} Ω, through which flows a current I=ΔV/R=0.73pA

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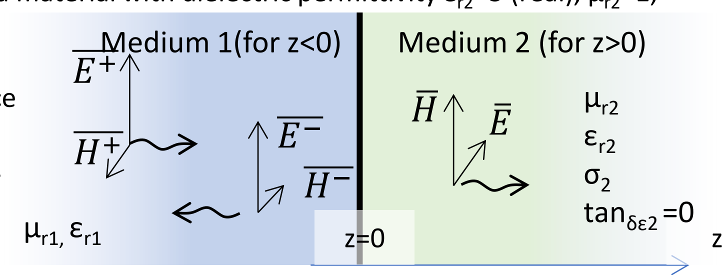

Solution

For a +z-travelling plane wave, the electric and magnetic fields are orthogonal to z and orthogonal to each other. Their cross product must be directed along +z.
The vectors in z=0, in phasor form, can be written as $E^+=E^+_x\ \hat{\imath}_x$ and $H^+=H^+_y\ \hat{\imath}_y$, so that the real Poynting vector S^+=1/2Real(E^+ x H^{+*}) is a positive quantity along $\hat{\imath}_z$ (power density carried by the wave)
In such a setting, the electric and magnetic phasor components are related by the impedance of the medium η: $E_x/H_y= \eta_1$. The wave impedance can be computed in general from its expression depending on the complex propagation constant $\gamma_1=\alpha_1$+j β_1: η_1=j ω μ_1/γ_1. The propagation constant is given by:
γ_1=sqrt((σ_1+j ω ε_1)j ω μ_1). The medium 1 is lossless, hence γ_1=j ω sqrt(μ_1 ε_1), α_1=0 and β_1=ω sqrt(μ_1 ε_1).
The expression of the impedance thus becomes: η_1=sqrt(μ_1/ε_1). Since $\eta_1=E_x/H_y$=266 Ω, $\varepsilon_{r1}=\mu_0/\varepsilon_0/\eta_1^2$=2.
Finally, β_1=740.98rad/m, from which λ_1=2/π/β_1=8.5mm.

The material in z>0 is lossy, due to the non-zero conductivity.
To be a good dielectric, the loss terms must be negligible: σ_2<<ωε_2. In this case ωε_2 =4.17, thus the good dielectric approximation is not valid.
Conversely, the conductive term dominates: σ_2 >>ω ε_2, so the good conductor approximation can be used.
In an ideal conductor (with σ=infinity), electrons move almost freely and thus react to any applied electric field to make it zero in the metal itself. As a consequence, the total electric field on the surface of the conductor is forced to be zero. Therefore, for a plane wave impinging on a good conductor one can expect a reflected wave with similar amplitude to the impinging one but opposite phase of electric field in z=0.
For a good conductor, the approximation states: γ_2~sqrt(jω$\mu_2\sigma_2$)=(1+j)/sqrt(2) sqrt(ω$\mu_2\sigma_2$) =3848 (1+j) 1/m, from which α_2=3848Np/m=33.4dB/mm and β_2=3848rad/m.
The skin depth is defined as an equivalent penetration length into the material, beyond which the fields are very small. Due to the exponential decay of a wave in a lossy medium $\exp^{-\alpha z}$, the skin depth is found as $\delta_2=1/\alpha_2$=0.26mm, coinciding with the expression δ_2=sqrt(2/(ω μ_2 σ_2)).
The wave impedance in the medium 2 is also approximated as a good conductor:
η_2=(1+j)sqrt(ω μ_2/2/σ_2)=25.6(1+j) Ω .
The reflection coefficient is: Γ = (η_2-η_1)/(η_2+η_1)=-0.81 + 0.159j=0.826 ∠169° (if Medium 2 were PEC, Γ=-1).

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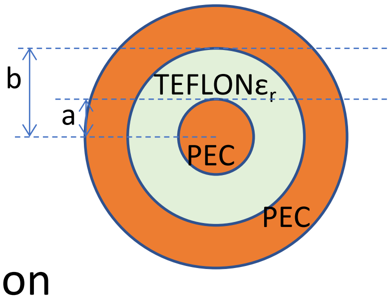

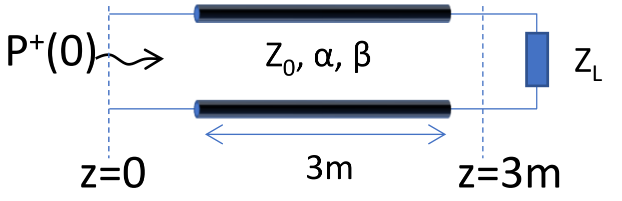

Solution

L can be immediately computed using the provided formula: L=219.7nH/m.

C can be obtained by using the relation for TEM lines: LC=μ ε: C= μ ε/L=103pF/m

R takes into account the power lost due to losses in the conductors due to finite conductivity. In this case the conductors are PEC, thus their conductivity is infinite and thus R=0 Ω/m.

G models the losses due to a leakage between the conductors due to a non-zero conductivity (or non-zero loss tangent) of the dielectric. Both in this case are 0, hence G=0 S/m.

The characteristic impedance of this lossless line is thus Z_0=sqrt(L/C)=46Ω.

The attenuation constant includes the effect of R and G, which are both 0. Thus α=0Np/m=0dB/m.

The phase constant for this low-loss line is β=ω sqrt(LC)=538rad/m=30825°/m=30.8°/mm.

The real power carried by the wave is $P^+(z)=|V^+(z)|^2/2/Z_0$, thus $|V^+(0)|$=214.5V. The phase of $V^+(0)$ can be arbitrarily chosen. The amplitude of the wave $|V^+(z)|$ remains constant since α=0.

The reflection due to a voltage reflection coefficient Γ_L at the end of the line creates a counter-propagating wave $V^-(z)$ with $V^-(3m)=\Gamma_L\,V^+(3m)$. The reflection coefficient is $\Gamma_L=(Z_L-Z_0)/(Z_L+Z_0)$=-0.642 and choosing the phase of $V^+(3m)$ as 0°, thus $V^-(3m)$=-137.7V.

The amplitudes of the propagating and counterpropagating waves are preserved along the line, since α=0. Thus the two waves combine as: $V(z)=V^+(0)\,e^{-j\beta z}+V^-(0)\,e^{+j\beta z}$. Thus for some z they combine in phase and for other z they combine in opposition, creating a voltage standing wave ratio (VSWR). Where they are in phase the total voltage is maximum and equal to $|V^+(0)|+|V^-(0)|$=352V.