POLITECNICO MILANO 1863 Academic year 2022/2023	094784 **Fund. of Electromagnetic Fields**	Date: ______________________
	Last Name:____________________	First Name: _________________
	Student ID ("Persona" or "Matricola"): ______________________________	Signature: ___________________________

30 August 2023 EXAM

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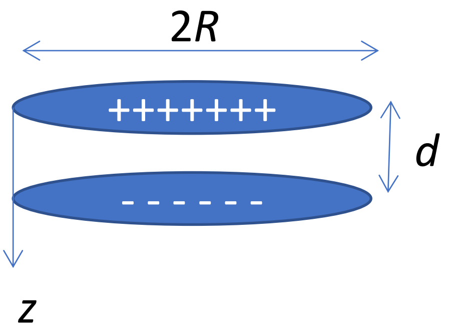
Exercise 1 [8 points]

A parallel plate capacitor with circular plates with radius of R=3cm spaced d=2mm has an electric field of 3kV/m constant between the plates.. The capacitor uses air as dielectric. Fringing fields at the border can be neglected.



1. Compute the potential difference ΔV between the plates
2. Compute the surface charge density on each of the plates, assumed constant.
3. A negative electrostatic point-like charge of q=-10pC and mass 1mg is inserted into such capacitor very close to the negatively-charged plate and thus feels a force. The charge is free to move. Determine the time it takes to reach the opposite plate, starting from being still. Remember that $F(N)=m(kg) \cdot a\ (m/s^2)$.

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Exercise 2 [12 pt]

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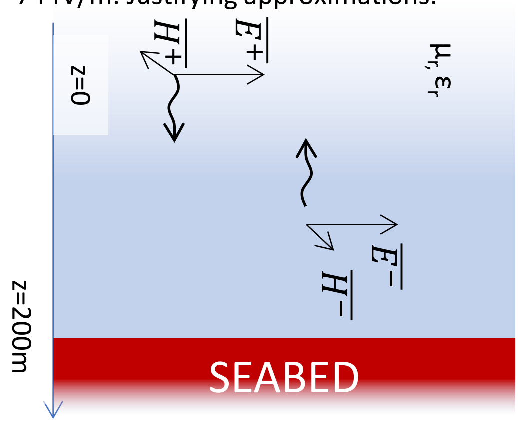

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1. If a series rectance X_M is inserted in series between the generator and the load (central figure), what value should it be to maximize the amount of power dissipated by the load (conjugate matching)? What is the amount of power which is absorbed by the load in this condition?
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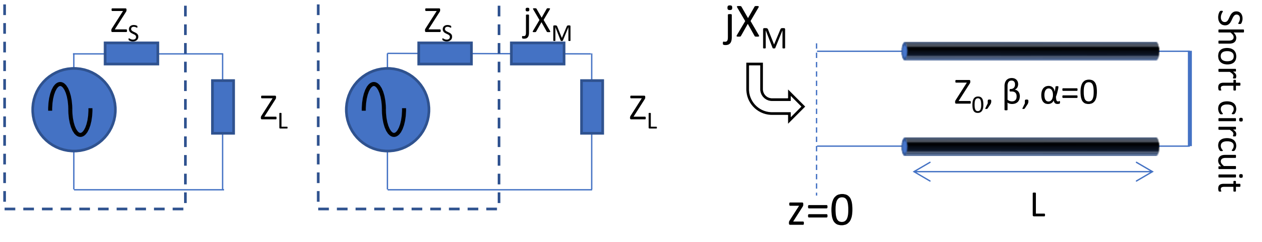

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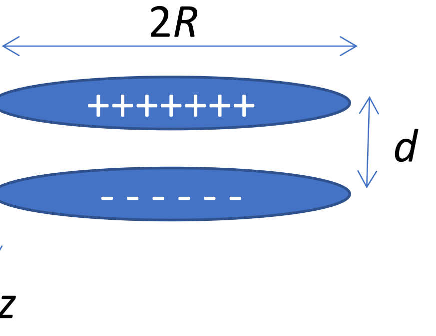

Solution

The electrostatic potential between the two plates is ΔV=Ed. The derivation is (not required):

$$\Delta V_{\{-Q\to+Q\}} = -\Delta V_{\{z=0\to z=d\}} = \int_0^d E\hat{\imath}_z \cdot \hat{\imath}_z dz = Ed$$

Hence ΔV=6V (between the positively-charged plate and the negatively-charged plate).

The capacitance between two charged bodies (Q and –Q), by definition, is C=Q/ΔV.
In a parallel plate capacitor the electrostatic field is constant in the whole capacitor and is directed away from the positively-charged plate: E $\hat{\imath}_z$
For a parallel plate capacitor with circular plates the formula is:
C=π R^2 ε/d =12.5pF. The amount of charge on each plate is thus Q=ΔV · C=75pC.
Assuming this charge to be uniformly distributed, the surface charge density is ρ_S=Q/(πR^2)=26.5nC/m^2.

Derivation (not needed):
Applying Gauss theorem to a cylindrical surface enclosing only the positive plate:

$$\oiint_S \overline{D}\cdot\overline{dS} = \iiint_V \rho_V dV \quad \to \mathrm{D} \int_0^R \int_0^{2\pi} r d\varphi\, dr = D\frac{R^2}{2}2\pi = DR^2\pi \quad = Q \ \to E = D/\varepsilon = Q/(\pi R^2 \varepsilon)$$

ΔV=Ed=Qd/(πR^2 ε) → C=Q/ΔV=(Qπ R^2 ε) /(Qd)=π R^2 ε/d

The force to which a charge q is subject is proportional to the electrostatic field: F=qE=30nN
The negative charge thus is subject to a force pushing from the negative plate toward the positive plate. Neglecting gravitational force, the corresponding acceleration is a=F/m=3·10^{-2} m/s^2. The stationary charge hence begins accelerating toward the positive plate, travelling a distance a · t^2, which requires t=sqrt(d/a)=0.258s.

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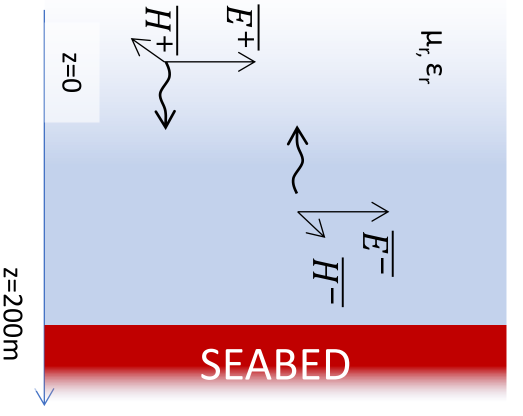

Solution

Check if water is a good dielectric: ωε >> σ? 2.25 · 10^{-7} S/m<<0.05S/cm=5S/m → Good conductor.
For a good conductor: γ =sqrt(j ω μ (j ω ε + σ)) ≈sqrt(j ω μ σ)=sqrt(ω μ σ/2) (1+j)
Thus: α=β= sqrt(ω μ σ/2)=0.0314 · m^{-1}. β=imag(γ)=0.0314rad/m=1.8°/m. α=real(γ)=0.0314Np/m=0.273dB/m.
Wavelength λ=2 π / β=200m. Characteristic impedance η_{WATER}= j ω μ/ γ =(1+j) sqrt(ω μ/2/σ)=6.28 (1+j) mΩ.
Assuming the electric field to be directed along x and the magnetic field along u, then, the wave propagates according to: $E^+_x(z)=E^+_x(0)\, e^{-\gamma z}$. Thus the amplitude of electric field decreases with:
$|E^+_x(z)|=|E^+_x(0)|\, e^{-\alpha z}$, thus giving $|E^+_x(200m)|$=1.39V/m. The corresponding magnetic field is orthogonal (in space) to the electric field and it is determined by the characteristic impedance: H^+_y(200m)=E^+_x(200m)/η=156A/m∠-45°, where the electric field is assumed with phase 0 at 200m.
The corresponding power density is obtained from the real Poynting vector, which has only z component:
S_z^+(200m)=0.5Real(E^+_x(200m) · (H^+_y(200m)*))=real(76.7-j76.7)W/m2=76.7 W/m^2.

The reflection coefficient of the electric field from water to seabed is:
Γ=(η_{SEABED}-η_{WATER})/(η_{SEABED}+η_{WATER})
The seabed is also a good conductor (ω ε =2.8^{-8} S/m << 1/rho=0.5S/m), thus the same approximated formula can be used for η_{SEABED}=(1+j) sqrt(ω μ/2/σ)=19.9(1+j)mΩ.
Thus Γ=0.52 (real).

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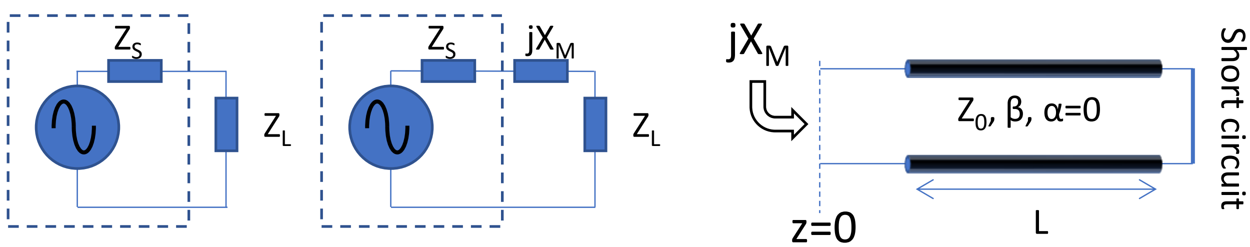

Solution

Maximum power transfer is obtained when the load (Z_L+jX_M) is conjugately matched to the generator impedance (Z_S): Therefore, we need X_M=-30Ω to cancel out the imaginary part of Z_L.
Under this condition, the real power absorbed by the load is maximum, and equal to the available power from the generator: PL= Pav=$(10V)^2/(8\ \text{real}(Z_S))$=0.25W.

The impedance observed from the input of a line is $Z_{in}=Z_0\ (1+\Gamma_L e^{-2j\beta L})/\ (1-\Gamma_L e^{-2j\beta L})$ when the other end of the line is terminated by a reflection coefficient Γ_L.
In our case, the line is short circuited, i.e. Z=0. The corresponding reflection coefficient is thus: $\Gamma_L=(Z-Z_0)/(Z+Z_0)=-1$.
Then, the derivation of the input impedance (not required) is:
Propagation and counterpropagation of two waves on the line follow $V(z)=V^+(0)\ e^{-j\beta z} + V^-(0)\ e^{+j\beta z}$ and $I(z)=I^+(0)\ e^{-j\beta z} + I^-(0)\ e^{+j\beta z}$ with $V^+(0)/I^+(0)=Z_0$ and $V^-(0)/I^-(0)=-Z_0$.
The ratio between the counterpropagating and propagating voltage wave is $\Gamma(z)= V^-(0)\ e^{+j\beta z}/(V^+(0)\ e^{-j\beta z})=\Gamma(0)\ e^{+2j\beta z}$
In our case, we know the reflection coefficient on the load $V^+(L)/V^-(L)=\Gamma(L)=\Gamma_L \rightarrow \Gamma_{in}=\Gamma(0)=\Gamma_L\ e^{-2j\beta z}$
The corresponding impedance is $Z_{in}=V(0)/I(0)=Z_0\ (1+\Gamma_{in})/(1-\Gamma_{in})$.

Therefore the input impedance, in our case, is $Z_{in}=Z_0\ (1-e^{-2j\beta L})/\ (1+e^{-2j\beta L})\ =Z_0\ j\tan(\beta L)$
We need Z_{in}=-j30 →tan(βL)=-30/60=-0.5 → βL=-26.6° + N 180°=153° → L=153° π /180°/β =5.45mm

The wavelength is λ=2 π / β =12.82mm at 16GHz.
Since the line is lossess (α=0), resistance R and conductance G per unit length are both 0 ($\alpha=R/2/Z_0+GZ_0/2$).
Moreover, we know β =ω sqrt(LC) and Z_0=sqrt(L/C) → β Z0=ωL → L=292nH/m; $C=L/Z_0^2$=81.2pF/m